Redirected Placement: Evaluating the Redirection of Passive Props during Reach-to-Place in Virtual Reality

Xuanhui Yang
Shanghai Jiao Tong University
Shanghai, China
yxh-v-7@sjtu.edu.cn

Yan Zhang
Shanghai Jiao Tong University
Shanghai, China
yan-zh@sjtu.edu.cn

Xubo Yang*
Shanghai Jiao Tong University
Shanghai, China
yangxubo@sjtu.edu.cn

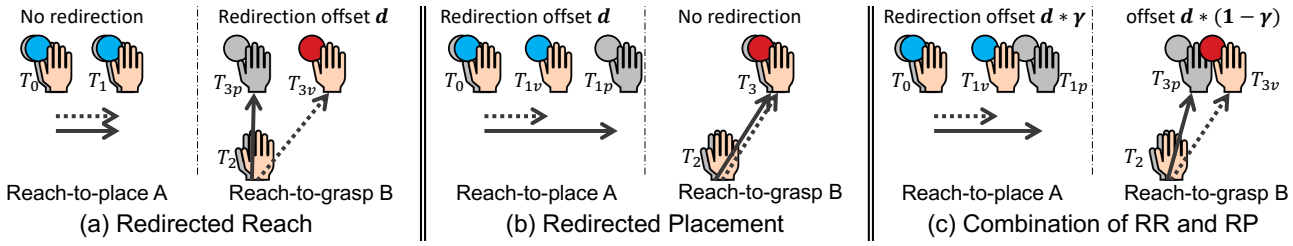

Figure 1: Redirected Reach (RR), the proposed Redirected Placement (RP), and the combination of RR and RP (RR & RP). Multiple virtual objects can be represented using a single physical prop by redirecting hand movements. RR modifies the virtual hand while grasping an object ("reach-to-grasp"), RP applies an offset after grasping an object ("reach-to-place"), and RR & RP combines both, with γ adjusting the redirection ratio between phases. Our experiment is designed with a 1-to-2 mapping from the physical prop (the gray circle) to two virtual objects, A (the blue circle) and B (the red circle). The positions of the virtual and real hands at various moments are marked as $T_0 - T_3$, with subscripts v and p used to distinguish them when not co-located. The virtual and physical trajectories are shown by dashed and solid arrows, respectively.

ABSTRACT

Hand redirection is an effective technique that can provide users with haptic feedback in virtual reality (VR) when a disparity exists between virtual objects and their physical counterparts. Psychophysiological research has revealed the distinct motion profiles of different kinematic phases when people operate hand-object interaction. In this paper, we proposed the Redirected Placement (RP), which determines the new placement of a physical prop using a constrained optimization problem. The visual illusion is used during the "reach-to-place" kinematic phase in the proposed RP method rather than the "reach-to-grasp" phase in the typical Redirected Reach (RR) method. We conducted two experiments based on the proposed RP method. Our first experiment showed that detection thresholds are generally higher with the proposed method compared to the RR method. The second experiment evaluated the embodiment experience with hand redirection using RR-only, RP-only, and RR&RP methods. The results report an enhanced sense of embodiment with the combined use of both RR and RP techniques. Our study further indicates that a 1:1 combination ratio of RR&RP resulted in the closest subjective experience to the baseline.

*Corresponding author.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than the author(s) must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from permissions@acm.org.
VRST 2023, October 09–11, 2023, Christchurch, New Zealand
© 2023 Copyright held by the owner/author(s). Publication rights licensed to ACM.
ACM ISBN 979-8-4007-0328-7/23/10…$15.00
https://doi.org/10.1145/3611659.3615688

CCS CONCEPTS

• **Human-centered computing** → **Virtual reality**; **User studies**.

KEYWORDS

Virtual Reality, passive haptic feedback, hand interaction, hand redirection

ACM Reference Format:
Xuanhui Yang, Yan Zhang, and Xubo Yang. 2023. Redirected Placement: Evaluating the Redirection of Passive Props during Reach-to-Place in Virtual Reality . In *29th ACM Symposium on Virtual Reality Software and Technology (VRST 2023), October 09–11, 2023, Christchurch, New Zealand*. ACM, New York, NY, USA, 11 pages. https://doi.org/10.1145/3611659.3615688

1 INTRODUCTION

In recent years, virtual reality (VR) has enabled users to experience realistic interactions with virtual objects in increasingly immersive virtual environments. In addition to visual and auditory sensations, haptic feedback is essential to improve users' sense of presence in a variety of VR applications. One of the methods to provide users with a realistic sense of touch is using passive haptics with hand redirection. By utilizing visuo-haptic illusions, hand redirection techniques extend the 1-to-1 mapping of physical objects (also called physical props) to virtual objects to a 1-to-n mapping and make the discrepancies of the position, size, or weight between virtual and physical objects unnoticeable, guaranteeing the versatility of the passive haptic system with various environment infrastructure [1, 5, 22, 30].

Manipulation of multiple objects typically occurs as a sequence of actions rather than in isolation in both real-world and VR tasks. For instance, when pouring water from a teapot into a cup, a person may first release the teapot to put it in the intended location

(the "reach-to-place" phase), then reach for the cup to grasp it (the "reach-to-grasp" phase). While both "reach-to-grasp" and "reach-to-place" phases can be utilized for hand redirection techniques to enable a 1-to-n mapping of physical and virtual objects in a passive haptic system, existing research primarily focuses on hand redirection in the former phase and the latter phase received less attention. However, previous research suggests that the sensorimotor control of humans during object manipulation follows distinct patterns in different kinematic phases [6, 10]. This implies that users' perceptions of redirection in different phases may differ. Therefore, it is crucial to evaluate hand redirection during different kinematic phases to support natural and sequential human object-directed actions in the passive haptic system.

The detection threshold of the misalignment between the physical props and the targeting virtual objects is crucial for building a robust hand-redirection-based passive haptic system. A few works have revealed the varied detection threshold while implementing hand redirection with different hand trajectory models[20], application scenarios[7], translation directions[2, 7, 19, 38], and avatars[27]. However, the detection threshold in the different kinematic phases of users is less explored. Research on object manipulations in VR indicated differences between the "reach-to-grasp" and "reach-to-place" phases concerning peak velocity and duration of the deceleration phase [23]. Very recently, Feick et al. revealed the correlation between the individual detection threshold and the velocity profile in VR hand redirection applications [9]. Therefore, it is reasonable to consider the distinct detection thresholds in the "reach-to-grasp" and "reach-to-place" phases for operating the hand-redirection-based passive haptic system efficiently.

Furthermore, existing methods for optimizing the redirection-based passive haptic system focus on the "reach-to-grasp" phase. To date, the exploration of the trajectory model during reaching virtual targets and the combination of hand redirection and other interaction techniques have improved the system performance significantly [1, 14, 30, 34]. Nevertheless, the utilization of hand redirection in the "reach-to-place" phase for the passive haptic system lacks investigation. An optimized strategy for implementing hand redirection in different kinematic phases according to the varied detection threshold is essential to provide users with a smoother interaction experience in VR.

To address this gap, we propose *Redirected Placement* (RP) in this paper. Different from methods that implement visuo-haptic illusions during the "reach-to-grasp" phase (e.g., *Redirected Reach* (RR)[30]) of object manipulation tasks (Figure 1 a), the proposed RP method is performed during the "reach-to-place" phase (Figure 1 b). We conducted two experiments to investigate differences in hand redirection between the "reach-to-grasp" and "reach-to-place" phases regarding the detection threshold and sense of embodiment.

This paper presents the following contributions:

(1) We propose *Redirected Placement* (RP) that operates the visuo-haptic illusion during the "reach-to-place" phase, rather than the "reach-to-grasp" phase for *Redirected Reach* (RR), in hand-redirection applications.
(2) We conduct an experiment to measure the detection thresholds of the "reach-to-grasp" phase and the "reach-to-place" phase in the object manipulation task with hand redirection. The result indicates a statistically higher detection threshold for RP operating in the "reach-to-place" phase.
(3) We conduct an experiment to evaluate the hand-redirection-based passive haptic application using RR-only, RP-only, and RR&RP, respectively. The result suggests a better sense of embodiment with the RR&RP case.

2 RELATED WORK

2.1 Kinematic Phases in Hand-Object Interaction

Aimed movements are common in object manipulation tasks and can be divided into a series of action phases. Each phase is bounded by the making or breaking of contact between the fingers and the object[10], such as reaching for an object and moving to place an object at a target location. Existing researches have confirmed various distinctions between different kinematic phases in both the real and the virtual environments. Studies in vision and physiology suggested the "reach-to-place" phase requires extended areas of the visual field compared to the "reach-to-grasp" phases, and vision problems influence the participants' performance of object manipulation in the "reach-to-grasp" and "reach-to-place" phases differently [28, 35]. Moreover, the object size and the target position also lead to the variation in the velocity profile and the movement profile of hand interaction [28]. Recently, Maldonado et al. proved that the kinematics of reach-to-grasp and reach-to-place movements in VR are also significantly different, as modulated by cognitive factors [23].

To date, a variety of hand redirection methods have been proposed for hand-object interaction in VR. However, existing works focus on the "reach-to-grasp" or "reach-to-touch" phases, with the application scenarios to point in mid-air [38], touch an interface [25], touch the other hand [8], touch a stationary object [1], or grasp an object [18]. Implementing hand redirection in the "reach-to-place" phase, such as placing a virtual object in a specific location, is rarely studied. Considering the kinematic differences between different kinematic phases (specifically the "reach-to-grasp" phase and the "reach-to-place" phase in this paper) is essential for using hand redirection techniques in VR applications efficiently.

In this paper, we propose the RP method that implements hand redirection in the reach-to-place phase. The experiments demonstrate that the RP method is as efficient as the typical RR method that operates in the reach-to-grasp phase of the hand-redirection-based passive haptic application.

2.2 Detection Threshold for Hand Redirection

Hand redirection enables physical props to represent virtual objects distinct from their position [1, 5, 8, 33], shape [3, 24, 36, 37], or weight [31]. Existing research suggests that people are more sensitive to visual cues than other sensory cues when they get several stimuli from various senses, termed the human visual dominance phenomenon [4, 13]. Besides, the illusion of the object's position in hand redirection is demonstrated to be less noticeable when it keeps coherent with the users' prediction based on the visual input, even though the direction or speed of hand movements differs between virtual and physical [10, 16]. Therefore, the hand redirection

technique is integrated into the passive haptic application to improve the system's versatility by mapping the same physical prop to multiple virtual objects [18, 30, 34].

Nevertheless, it is also proved that people have limited tolerances of the position and velocity illusion in the hand redirection application. Users no longer accept the visuo-haptic illusion when the co-location or similarity criteria between physical and virtual objects exceed a certain threshold [26]. Therefore, investigating the detection threshold is critical for building a robust hand redirection system. A few works quantified the just detectable misalignment between the physical prop and the mapped virtual objects for hand redirection, revealing that the detection threshold is not a fixed range but is affected by many factors, such as movement directions [2, 11, 38], complexity of the scenes [7], appearances of the avatars [19, 27], interpolate functions [12, 20], and number of the hands involved [15]. In addition, the detection threshold is suggested to be redirection-approach-related. Han et al. evaluated the detection threshold of hand redirection using translational shift and interpolated reach, respectively [18]. Their results show that both remapping methods are equally effective when there is a slight mismatch (10-14 inches) between virtual and physical objects, and the translational shift outperformed the interpolation reach when the mismatch becomes significant (30 inches). Patras et al. compared between body warping and change blindness remapping for redirecting the hand to the horizontally distributed virtual buttons [29]. The results report a considerably higher detection threshold (9.7cm) with the change blindness than the body warping (7.9cm). Feick et al. recently utilized physiological data and kinematic interaction data to reflect the user's inconsistency during hand redirection applications, demonstrating that the detection threshold varies for many factors and even from person to person [9].

Although the detection threshold is evaluated with various variables in hand redirection, the influence of the kinematic phase on the threshold is less explored. We conduct experiments to assess detection thresholds in the "reach-to-grasp" phase and the "reach-to-place" phase in the object manipulation task in VR. The experiment result suggests different thresholds in the different kinematic phases.

2.3 Optimization for Hand Redirection

Since the limited detection threshold restricts the performance of hand redirection methods, a variety of optimized methods are proposed by the state-of-the-art works, demonstrating the improved performance in VR applications.

Lebrun et al. examined a hand trajectory model based on cubic Bézier curves with four control points to reveal the relationship between the amplitudes of redirection and hand trajectories [20]. Their trajectory model is easily refined to provide undetected redirection for each individual by adjusting a parameter that describes their reaching patterns. By replacing the linear interpolation method in redirection with a second-order redirection equation, Geslain et al. [12] provided more flexible redirection designs that can control the trajectory of the hand and avoid collisions. A few works combined hand redirection techniques with dynamic passive haptic devices to complement the limitations of either technique and to extend the noticeable mismatch between physical and virtual environments [14, 39, 41]. In the study of Sait et al. [30], Redirected Reach was utilized together with resetting or guided rotation techniques to guarantee users interact with multiple virtual objects through a single physical prop in a comfortable interaction zone. Zenner et al. [40] explored the combination of continuous redirection with blink-suppressed instantaneous shift in different ratios. The psychophysical experiments they conducted indicated that the detection thresholds varied based on the different ratios, and the combined method could be less noticeable than using pure blink-suppressed redirection.

Distinct from the existing research that optimizes hand redirection by using better interpolation algorithms or combining it with other interaction techniques, we suggest improving hand redirection by integrally implementing the visuo-haptic illusion in both the "reach-to-grasp" and the "reach-to-place" kinematic phases. Both the subjective result and the objective data in our experiment suggest a better sense of embodiment when using RR and RP together with a ratio of 1 : 1.

3 REDIRECTED PLACEMENT

3.1 Target Selection for the Physical Prop in RP

With hand redirection techniques like Haptic Retargeting [1] and Redirected Reach [30], one physical object can provide haptic feedback for multiple virtual objects separately so that they improve the versatility of the haptic system. Different from existing methods that operate hand redirection in the "reach-to-grasp" phase, the proposed RP method works during the "reach-to-place" phase. Redirection algorithms for RR systems cannot be applied directly to the RP system because the redirection target in the "reach-to-place" phase is uncertain. Therefore, to warp the user's hand trajectory during the "reach-to-place" phase, one critical point is determining the target location for redirection. Accordingly, we simplify the target selection problem to specify the placement location of the virtual object and use a constrained optimization problem to find the target location of the physical prop. The solved location minimizes the offset between the physical prop and the virtual objects while keeping the redirection within the detection threshold.

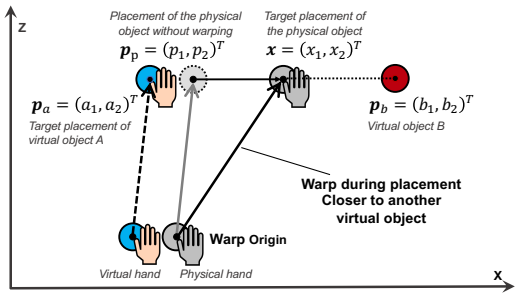

Figure 2: *Redirected Placement*: warp the virtual hand (the colored one) while placing the virtual object A (the blue circle) to adjust the placement of the physical object (the gray circle) closer to the other virtual object B (the red circle).

To introduce the proposed RP algorithm, we focus on a simplified setup consisting of two virtual objects, A and B, and one

Algorithm 1 Pseudo code of Preparation for Redirected Placement

Input: $\boldsymbol{p}_{a_0}$: current position of the virtual object A;
$\boldsymbol{p}_b$: current position of another virtual object B;
$\boldsymbol{p}_{p_0}$: current position of the physical prop mapped to A;
θ: detection threshold of redirected placement;
ω_1, ω_2: constant weights.
Output: $\boldsymbol{p}_x$: The updated target placement of the physical prop.

$\boldsymbol{offset} \leftarrow \boldsymbol{p}_{p_0} - \boldsymbol{p}_{a_0}$ ▷ current offset
$\boldsymbol{p}_a \leftarrow GetVirtualDestination(\boldsymbol{p}_{a_0})$ ▷ specified placement
$\boldsymbol{p}_p \leftarrow \boldsymbol{p}_a + \boldsymbol{offset}$ ▷ origin physical placement
$\boldsymbol{x}_0 \leftarrow \omega_1 \boldsymbol{p}_a + \omega_2 \boldsymbol{p}_b$ ▷ interpolation point
if $Distance(\boldsymbol{x}_0, \boldsymbol{p}_p) \leq \theta$ **then** ▷ interior solution
    $\boldsymbol{p}_x \leftarrow \boldsymbol{x}_0$
else ▷ boundary solution
    $\boldsymbol{p}_x \leftarrow \boldsymbol{p}_p + \theta \cdot Normalize(\boldsymbol{x}_0 - \boldsymbol{p}_p)$
end if
return $\boldsymbol{p}_x$

physical prop, P. The interaction is assumed to pick up A, move to the specified location to place it, and then manipulate B. In this process, the same physical prop P represents both A and B. Figure 2 illustrates how the RP method gradually modifies the virtual hand movement to change the placement of the physical object in the "reach-to-place" phase. The target location for placing the physical prop in the real world after picking up A can be calculated based on the specified placement of the virtual object A and the location of another virtual object B in the virtual world. The proposed solution aims to minimize the offsets between the physical prop P and the virtual objects A and B. Additionally, the warped distance is subjected to the detection threshold during the interaction. This can be formulated as follows:

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \frac{1}{2}\omega_1 \|\boldsymbol{x} - \boldsymbol{p}_a\|_2^2 + \frac{1}{2}\omega_2 \|\boldsymbol{x} - \boldsymbol{p}_b\|_2^2 \\ \text{subject to} \quad & \|\boldsymbol{x} - \boldsymbol{p}_p\|_2^2 \leq \theta^2 \end{aligned} \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2)^T$ represents the updated target placement of the physical prop, while $\boldsymbol{p}_p = (p_1, p_2)^T$ is the original placement of the physical prop. Virtual objects A and B are placed at $\boldsymbol{p}_a = (a_1, a_2)^T$ and $\boldsymbol{p}_b = (b_1, b_2)^T$ respectively. θ is the detection threshold of the "reach-to-place" phase, which will be evaluated in the following user study. ω_1 and ω_2 are two constant weights used to control the ratio of the physical object's distance to the two virtual objects, following the constrain of $\omega_1 + \omega_2 = 1$. The detailed solving procedure of this optimization problem can be viewed in Appendix A.

Alg. 1 describes the process of updating the placement position of the physical prop based on the current positions of the objects. As we specify where to place the virtual object A, the updated placement of the physical prop can be determined by taking into account the current positions of the physical prop ($\boldsymbol{p}_{p_0}$) and virtual objects A ($\boldsymbol{p}_{a_0}$) and B ($\boldsymbol{p}_b$).

3.2 Combination of RP and RR

Hand redirection requires the distance between the virtual object and the physical prop to be smaller than the detection threshold to avoid the reduction of the users' sense of embodiment in VR applications. Since the same physical prop may be mapped to multiple virtual objects during a series of manipulation tasks, it is reasonable to minimize the noticeable mismatches by combinedly implementing hand redirection in both the "reach-to-grasp" and the "reach-to-place" phases. Therefore, we further propose the combination of the typical RR method and the proposed RP method in the 1-to-2 mapped passive haptic application.

The constant weights ω_1 and ω_2 are assigned values in the range of $[0, 1]$ to control the combination ratio between RR and RP. In the cases where the weight of the first object A, which relates to ω_1, is assigned 1 or 0, the physical prop's designated placement during the "reach-to-place" phase matches the position of the first object or the second object, which results in the exclusive use of RR or RP, respectively, as shown in Figure 1 (a) and (b). Furthermore, by assigning a value within the range of $(0, 1)$ to ω_1, the hand redirection is sequentially executed during the "reach-to-place" phase of the first object and the "reach-to-grasp" phase of the second object , where ω_1 equals to $(1 - \gamma)$ in Figure 1 (c) . Similar to the existing dynamic redirection method [1], the proposed RP algorithm dynamically interpolates the hand's position after selecting the target. Hence, the hand redirection system with the typical RR methods can be easily integrated with the proposed RP method.

Firstly, the target placement of the physical prop $\boldsymbol{x}$ is updated based on the mentioned target selection function. So the target warping offset $\boldsymbol{w}_{rp}$ is:

$$\boldsymbol{w}_{rp} = \boldsymbol{p}_a - \boldsymbol{x}. \tag{2}$$

Then, RP applies the dynamic warping offset to the virtual hand in the "reach-to-place" phase. The offset $\boldsymbol{w}$ is computed as follows:

$$\boldsymbol{w} = \alpha \boldsymbol{w}_{rp}, \quad \alpha = \frac{D_s}{D_s + D_p}, \tag{3}$$

where D_s is the distance between the warping origin and the current position of the physical hand, and D_p is the distance between the physical hand and the target placing position. After the user places and releases from the object, the system gradually shifts the virtual hand to realign with the user's real hand during the hand-moving period. Subsequently, the same physical prop is used to represent the second virtual object, and RR is performed to compensate for the remaining offset between the virtual object and the physical prop.

The combination of RR and RP allows the misalignment between the virtual object and the physical prop to be compensated by redirecting in two movement phases. As a consequence, each redirection can remain unnoticeable even when the overall misalignment exceeds the detection thresholds of each sub-phase.

4 EVALUATION

4.1 Apparatus

The experimental setup is shown in Figure 3. Both physical and virtual environments consisted of a table ($90cm \times 50cm$) with cylindrical objects ($\phi 6.5cm$) on the table. We used an OptiTrack system with four Prime 13 cameras to track and locate the hands and objects, where we tracked all of the targets as rigid bodies through unique marker patterns. The participant can grasp and move the physical prop (the black rod) while simultaneously seeing the virtual hand move the virtual object (the red rod). Since the redirection

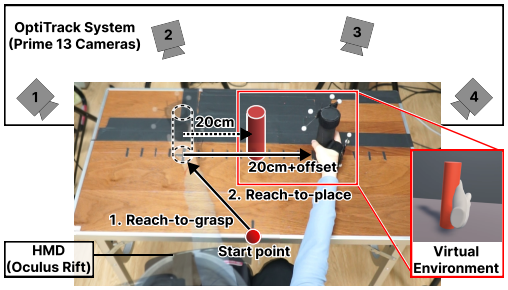

Figure 3: Top-down view of the experimental setup.

technique is used, the physical prop can be shifted from the location of the virtual objects. Both experiments were designed with a similar setup. The first experiment involved utilizing one interactable virtual object, while the second experiment used a 1-to-2 mapping that included two virtual objects with one physical prop. We implemented the experiments with the Virtual Reality Toolkit (VRTK)[1] in Unity 3D (2019.4.22f1c1) and ran them on the Oculus Rift HMD.

4.2 Experiment 1: Detection Threshold Estimation for Different Kinematic Phases

4.2.1 Design. The first experiment only used one interactive virtual object to focus on user acceptance of misalignment during a single interaction process. The participants were asked to grasp the virtual object ("reach-to-grasp" phase) and then move it to the specified location on the table ("reach-to-place" phase). We used a within-subjects design, and the independent variables in this experiment were the methods used for hand redirection (RR-only, RP-only, and RR&RP) and the redirection magnitude.

As illustrated in Figure 3, the interaction task is designed as moving the virtual object from the original location (the dashed rod) to the target location at the center of the table (the red rod). The moving distance of the virtual object was constant (d=20cm), and the physical prop was initially placed at 20cm to the left of the center of the desktop. For the RR-only and RP-only methods, the hand was redirected during the "reach-to-grasp" phase and the "reach-to-place" phase, respectively. Furthermore, the combination method applied both RR and RP in the same task, which executed the warping to the virtual hand during both the "reach-to-grasp" and "reach-to-place" phases. Each type of task was set up with six kinds of offsets (0, 4, 8, 12, 16, 20cm), performed in random order. In particular, the redirection offsets in the two kinematic phases were equal for the task with the combined method. Every task was repeated twice to increase the credibility of participants' answers. Therefore, each participant was required to complete 36 trials (3 types of the redirection method $\times$ 6 offsets $\times$ 2 repetitions). To avoid fatigue from long-term use of VR, participants took a break once they completed a repetition.

In this experiment, we conducted pseudo two-alternative forced-choice (pseudo-2AFC) tasks to estimate the detection threshold of redirection in each kinematic phase, which required participants to judge whether the virtual movement was the same as the physical one. 2AFC or pseudo-2AFC are commonly used psychometric designs for measuring a person's sensitivity by letting participants choose from two stimuli or Yes/No, and it is a typical analysis for estimating the threshold [27, 29].

As a consequence, we used the psychometric function to analyze the detection thresholds of redirection in different kinematic phases:

$$y = \frac{1}{1 + e^{-b(x-a)}}, \tag{4}$$

where x is the offset between virtual and physical objects, and y is the probability of considering the virtual movements sync with physical actions. The point $x = a$ is the 50% detection probability, representing the point of subjective equality (PSE), and b represents the associated standard deviation.

4.2.2 Participants. Nineteen participants (7 females, 12 males) volunteered to take part in our user study. The participants were all undergraduate or graduate students with an average age of 22.21 (SD=1.10), and all were right-handed and had normal or corrected-to-normal vision. Regarding their VR experience, 2 participants (10.53%) reported that they use VR frequently, 4 participants (21.05%) used VR occasionally, 7 participants (36.84%) used VR once or twice, and 6 participants (31.58%) never used VR. In addition, 10 of the total 19 participants also reported having at least once or twice VR development experience.

4.2.3 Procedure. Firstly, participants went through a set of training trials to get familiar with the operation in the experiment, which was to grasp and place the virtual object in the virtual environment. In the training session, we did not introduce any offset to the virtual hand so that participants could establish the correct perception of the virtual-real relationship when adapting to grasping virtual objects with haptic feedback in the virtual environment.

For each trial, the participants were asked to complete a task of picking up a virtual object and placing it at a specified location. Participants were able to see a panel reminding them what to do. They started the task by touching the red sphere in front of them. After the task started, the application randomly set a pre-defined offset to initialize the virtual object's position. Then, participants needed to reach to grasp the virtual object. When participants grabbed the object, a semitransparent cylinder would appear in the virtual environment, indicating the target location for placing the virtual object. Following each iteration, the participants were presented with a query, "I felt that the movement of the virtual hand was the same as my own movement," on the instruction panel in front of them in the virtual environment. Participants were required to select a response by utilizing the left controller. It is worth noting that participants were precluded from interacting with any objects or observing the virtual hands during the selection process to guarantee an impartial perception of the recent interaction.

4.2.4 Results.

Detection Thresholds. Firstly, we calculated the percentage of trial errors in the experiment based on the recorded trajectory data. Specifically, we defined an error as occurring when participants accidentally knocked over the physical object while attempting to grasp it by checking whether the physical prop appeared in

[1]VRTK. https://www.vrtk.io

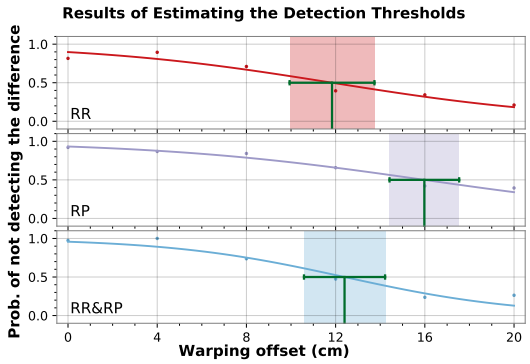

Figure 4: Fitted curves of the psychological equation for participants' feelings on 3 redirection methods (RR: Redirected Reach, RP: Redirected Placement, and RR & RP). The green vertical lines mark the detection thresholds with 95% confidence intervals.

Table 1: The detection thresholds for three hand redirection methods. Confidence interval(CI) indicates that the detection thresholds are within this range with 95% probability.

Method	Detection threshold [CI] (cm)
RR	11.84 [9.94, 13.74]
RP	15.98 [14.42, 17.54]
RR&RP	12.40 [10.58, 14.22]

locations not part of the actual hand movement path. The average error rate was 3.2% for each condition (N=38, 19 participants × 2 repetitions), which means that only 1-2 errors happened with each offset for each method.

We fit the function after excluding errors. Figure 4 displays the fitted curves of perceptual data from participants in three redirections conducted at various kinematic phases and offsets. We also identified the points in the figure that have a 50% probability of perceiving the virtual hand motion as consistent with the real hand motion. Additionally, Table 1 offers the value of detection thresholds with a 95% confidence interval. The results indicate that individuals have a greater tendency to disregard the misalignment between the virtual hand and the real hand during the "reach-to-place" phase ($DT_{RP} = 15.98 \pm 1.56cm$) compared to the "reach-to-grasp" phase ($DT_{RR} = 11.84 \pm 1.90cm$). Moreover, implementing redirection in both kinematic phases of a single task (RR&RP) closely aligned with the smaller detection threshold scenario (RR), which resulted in $DT_{RR\&RP} = 12.40 \pm 1.82cm$.

Interaction Data. We analyzed the interaction data of hand redirection with different offsets for each kinematic phase on the completion time, movement distance, and motion profiles breakdown into two sub-phases (ballistic phase and correction phase). The "reach-to-grasp" and "reach-to-place" phases are marked according to the time that the object starts moving. The sub-phases are divided by using the method introduced by Liu et al. [21]. Figure 5 shows

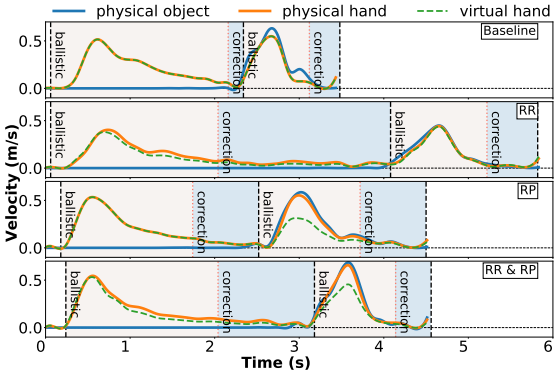

Figure 5: Velocity profiles of Participant-12. The first ballistic and correction sub-phases belong to the "reach-to-grasp" phase, and the later sub-phases are part of the "reach-to-place" phase. The baseline trial has zero offset, and offsets in RR, RP, and RR&RP trials are all 16 cm in this case.

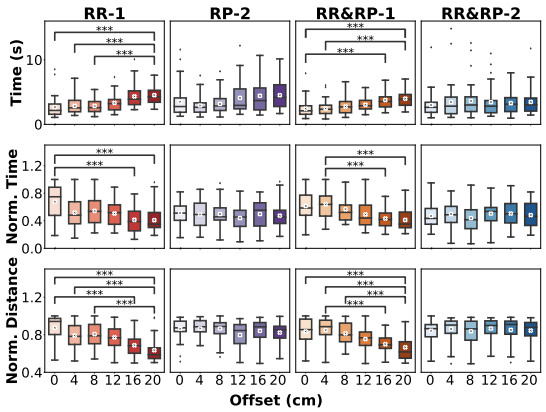

Figure 6: Completion time of the redirected phases (top), normalized time (middle), and normalized distance (bottom) of the ballistic sub-phase in different kinematic phases when using RR, RP, and RR & RP methods. Each method is implemented with different offsets from 0cm to 20cm. The "reach-to-grasp" and the "reach-to-place" phases are marked as the suffixes 1 and 2, respectively. $p < .001(***)$

the velocity profiles of participant-12 in four distinct trials with different redirection methods. The velocity profile of the hand during each trial exhibits two distinct peaks, corresponding to the velocity peaks of the "reach-to-grasp" and "reach-to-place" phases (separated by the black dashed line). The ballistic and correction sub-phases are highlighted with light skin and light blue backgrounds, respectively. The ratio of ballistic to correction was not the same across the kinematic phases, suggesting that different redirection settings affect participants' adjustment to their movements.

The recorded interaction data was evaluated with Shapiro-Wilk's normality test, and the result indicates a reliable normal distribution. As a next step, we conducted an ANOVA test to examine the differences in redirection offsets during the "reach-to-grasp" and

"reach-to-place" phases. After identifying significant differences with a p-value less than 0.05, we conducted post-hoc pairwise comparisons by using Scheffe's test. To distinguish between the kinematic phases and redirection settings, we assign the number 1 to the "reach-to-grasp" phase and the number 2 to the "reach-to-place" phase. For instance, "RR&RP-1" notation refers to the "reach-to-grasp" phase when using the hand redirection with the combined method.

The statistics results of the completion time and the normalized time and distance in the "reach-to-grasp" kinematic phase (with the notation of "1") are shown in Figure 6. Firstly, we found that the completion time of the "reach-to-grasp" phase varies significantly across the redirection offsets (RR-1: $F(5, 189) = 10.12$, $p < .001$; RR&RP-1: $F(5, 195) = 11.08$, $p < .001$). However, the differences in the "reach-to-place" phase are less noticeable (RP-2: $F(5, 193) = 4.27$, $p < .01$; RR&RP-2: $F(5, 195) = 0.64$, $p > .05$). As illustrated in the top row of Figure 6, the completion time with the redirected offset of 20 cm ($M_{20} = 4.62, SD_{20} = 1.53$) was significantly longer than the time with smaller offsets in "RR-1" (0cm: $M_0 = 2.67, SD_0 = 1.68$; 4cm: $M_4 = 2.85, SD_4 = 1.23$; 8cm: $M_8 = 2.86, SD_8 = 1.10$), while in "RP-1", only the case with the offset of 20 cm took statistically longer time than the baseline ($\Delta_{0-20} = -1.42$, $p < .05$).

Then, we further analyze the effect of redirection offsets on the ballistic and correction phases. We normalized the time and distance of each ballistic sub-phase by dividing the total time and distance of each kinematic phase. Therefore, a decrease in the normalized time or distance indicates a lower percentage of the ballistic phase and a higher percentage of the correction phase. As shown in the rest rows of Figure 6, the proportion of the correction sub-phase in the whole movement phase increases significantly in time and movement distance when the redirection offset reaches 16 cm and 20 cm in the "reach-to-grasp" phase. However, the differences are not obvious in the "reach-to-place" phase. Considering redirection offsets that exceed the detection threshold result in conflicting predictions and visions, it is reasonable for participants to spend more time correcting their hand movements, which aligns with the smaller detection threshold of the RR method than the RP method in Figure 4.

4.3 Experiment 2: Embodiment Evaluation

4.3.1 Design. We evaluated the sense of embodiment with various combination ratios of the RR&RP method in the second experiment. This experiment was conducted with another group of participants on a different day from the first experiment. Figure 7 illustrates the procedure for the second experiment. In the first reach-to-place phase (2), the movement distance was 20cm from left to right, which is consistent with our first experiment, while the movement was reduced to 10cm from right to left during the second reach-to-place phase (5) to prevent collision between two virtual objects. Based on the results of our first experiment, we chose the overall redirected offset (d) as 18 cm, which was large enough for participants to notice the discrepancy if hand redirection only works with RR or RP methods.

The independent variables in this experiment were the combination ratio between the "reach-to-place" phase of the first object and the "reach-to-grasp" phase of the second object. We set up five

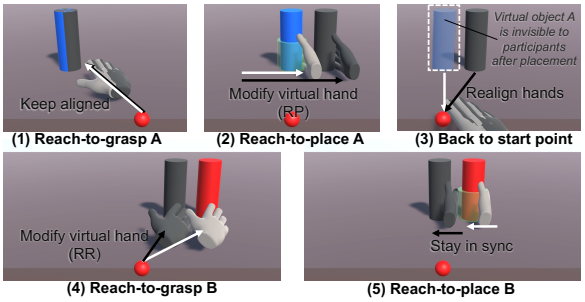

Figure 7: The procedure of Experiment 2. Utilizing the same physical object (the gray rod, which is invisible to participants in the experiment) as a haptic proxy for two virtual objects (the blue rod and the red rod) by combining RR and RP. The dark and slightly gray hands indicate the participant's real and virtual hands.

conditions with different combination ratios as well as the baseline condition. For the baseline condition, we used two physical props to apply 1-to-1 mapping for the two virtual objects. The overall offset of 18cm was applied with different combinations of RR and RP methods. The ratio was set as $(1:0)$, $(2:1)$, $(1:1)$, $(1:2)$, and $(0:1)$, where $(1:0)$ and $(0:1)$ represented RR-only and RP-only cases, and the others balance RR and RP methods with different weights. The different ratios of redirection offsets for the RR and RP method are achieved by correspondingly adjusting the ω_1 in section 3 when computing the target placement of the physical prop.

The combination ratio was randomly chosen in the experiment, and the arrangement guaranteed each condition with the unique ratios was implemented and only implemented once. The baseline condition was implemented twice and consistently introduced in the first and fifth trials. This allowed for evaluating any variances in the baselines resulting from the participant's familiarity with the task. Moreover, setting the first group of the formal experiment as the baseline condition can ensure the accuracy of the participants' calibration of the virtual environment.

In each condition, participants were required to complete 5 repeated trials. The reasons for setting this small number of repetitions are to control the duration of the entire user study from being too long, and to avoid users adapting to the abnormalities of redirection with repeated operations. Both the fatigue and adaption of participants might affect their performance in the experiment, hiding the differences in RR&RP from various combination ratios. After each condition, we used a subset of the questionnaire about embodiment [17] to evaluate participants' feelings about ownership, agency and motor control, and location of the hand, while using different ratios to combine redirection (Appendix B). Therefore, each participant was required to complete 35 trials (7 conditions (5+2) × 5 repetitions) and fill out the questionnaire 7 times.

4.3.2 Participants. A total of 18 individuals participated in the second experiment, comprising 5 females and 13 males. All the participants were students enrolled either in undergraduate or graduate programs, with an average age of 24 years (SD=1.94). They all had

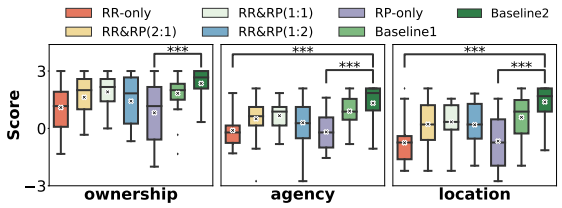

Figure 8: Mean ratings of ownership (left), agency and motor control (center), and location of the hand (right) for each redirection condition in Experiment 2 of the embodiment evaluation. $p < .001(***)$

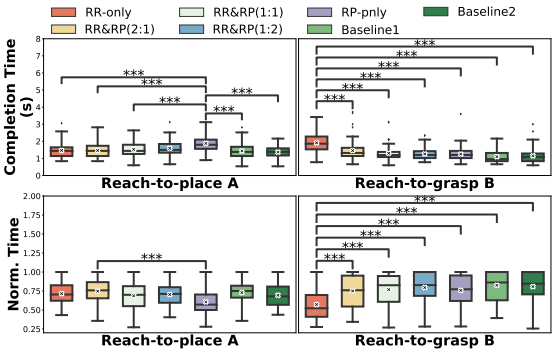

Figure 9: Completion time of the redirection phase (top), normalized time of ballistic phase (bottom) for each redirection condition, and kinematic phase. $p < .001(***)$

normal or corrected-to-normal visions and were right-handed. In addition, a vast majority of the participants (66.67%) had limited exposure to virtual reality, having experienced it only once or twice, and one participant reported having no prior experience with VR. About 22.22% of participants used VR 1-2 times per month, while only one participant had a weekly VR experience.

4.3.3 Procedure. Participants in the second experiment underwent a training phase similar to the first experiment to become familiar with the task procedure before the formal trials. During the formal experiment, after five repetitions of each condition, participants were asked to complete a 7-point Likert scale (from -3 to 3) questionnaire about embodiment outside the VR environment.

In each trial, the instruction panel in front of the table in the virtual experiment guided participants with the next interaction task. As shown in Figure 7, the virtual and physical environments were aligned so that without any warping in the first "reach to grasp" phase with the virtual object A. During the second or fourth steps, hand redirection was implemented during the interaction with virtual objects A and B, while RP and RR worked in the "reach-to-place" phase (2) and the "reach-to-grasp" phase (4), respectively. The redirection offsets in phases (2) and (4) were controlled to meet the specific combination ratios of RR and RP methods and keep the same summed offset of 18cm. Besides, a relocation step (3) was operated to prevent the influence of the offset in phase (4) due to the redirection used in phase (2).

4.3.4 Results.

Virtual Embodiment Questionnaire. The Kruskal-Wallis test is employed to analyze the differences among the various conditions for each questionnaire item. We conducted additional pairwise comparisons using the Dunnett test once considerable differences ($p < .05$) were found.

As shown in Figure 8, significant differences are observed between the redirection conditions and the baseline (1-to-1 mapping) regarding ownership ($H = 18.069$, $p < .001$), sense of agency ($H = 28.764$, $p < .001$), and location ($H = 32.022$, $p < .001$). Moreover, the combined redirection approach (RR&RP) indicates statistical differences in several aspects when compared to the RR-only and RP-only approaches, specifically in Q1, Q2, Q4, Q6, Q8, and Q9 of the questionnaire. Combining the scales of ownership, agency, and location, the median ranking of all conditions was: Baseline2 ($M = 2.57$) > Baseline1 ($M = 1.89$) > RR&RP($1:1$) ($M = 1.47$) > RR&RP($2:1$) ($M = 1.18$) > RR&RP($1:2$) ($M = 1.14$) > RR-only ($M = 0.24$) > RP-only($M = 0.18$). This result indicates that a balanced hand redirection method of implementing RR and RP in the "reach-to-grasp" and the "reach-to-place" kinematic phases makes the visual illusion less noticeable by consistently keeping the offset under the users' detection thresholds. Consequently, the combined implementation of RR and RP improves the sense of embodiment efficiently for hand-redirection-based passive haptic feedback applications.

Interaction Data. Similar to the first experiment, the recorded participant's movement data was divided into multiple sub-phases to analyze the characteristics of each movement phase. In this experiment, we analyzed four interaction phases with the two virtual objects A and B, labeled as "reach-to-grasp A," "reach-to-place A," "reach-to-grasp B," and "reach-to-place B." After conducting the Kolmogorov-Smirnov test, processed data follows a normal distribution. An ANOVA was employed to analyze variations across conditions, followed by post-hoc pairwise comparisons utilizing Scheffe's method.

Figure 9 shows that RR-only and RP-only conditions made participants take more time in the correction phase to get the "correct" position (A: $F(6, 505) = 4.77$, $p < .001$; B: $F(6, 505) = 12.13$, $p < .001$), and lead to a longer completion time (A: $F(6, 505) = 12.09$, $p < .001$; B: $F(6, 505) = 31.93$, $p < .001$). This reveals that the participants' predicted moving distance or reaching direction deviated further from the visual guidance with a large redirection condition. Nevertheless, no significant differences in task completion time were found among the conditions, and only the RR-only condition differed from the 1:1 combined condition ($\Delta_{RR-RR\&RP} = 0.947, p < .01$) and the baseline condition ($\Delta_{RR-Baseline} = 1.003, p < .01$) in terms of movement distance.

5 DISCUSSION

Less noticeable when placing than reaching. Both the detection threshold and the interaction data in the first experiment indicate that redirection at the "reach-to-grasp" phase is likely to be less noticeable to users. According to our observation, users are more likely to control hand movements according to their predictions when reaching to grasp the object. In contrast, when the target placement is specified, users probably rely more on vision to adjust

hand movements. Compared with the previous studies[29, 38], our estimated detection thresholds for Redirected Reach are slightly larger, probably because the tasks in our experiment that have participants pick up and place the object, which may distract them from kinesthetic perception. What is more, our first experiment also notes that the effective detection threshold is close to the minimum one of the single kinematic phase when implementing hand redirection for a multi-phase interaction. It appears that the interaction data for two continuous redirections is similar to separated kinematic phases, which may explain that any inconsistencies between the virtual and real environments can easily break the sense of embodiment during the interaction. Hence, it is essential for designers to carefully evaluate the potential effects of repeated redirections in interactive applications, taking into account both the scale of each redirection and its overall impact on ongoing interactions.

Combined redirection of various kinematic phases. In the second experiment, we discovered that utilizing all three conditions of RR&RP resulted in a superior embodiment experience for participants compared to solely using RR or RP. This is due to the fragmentation of the final redirection offset, which makes each redirection less overwhelming. Following the experiment, we conducted interviews with participants to gather their feedback. A few participants expressed difficulty touching the object and noticed a gap between their virtual hand and the object, which did not match their prediction. Additionally, some of them reported feeling perplexed after moving an unusually long distance when placing the first object. Moreover, the combined conditions performed well enough that some participants could not clearly distinguish them from the baseline. Some participants even reported that they did not perceive which specific case two physical objects were used among the various conditions. In the second experiment, the distance between two virtual objects exceeded the detection threshold for a single redirection, resulting in a poorer sense of embodiment. However, the RR&RP conditions perform an obviously better sense of embodiment for the same distance. The performance with the RR&RP methods is similar to the 1-to-1 mapping baseline condition by allocating the redirection offset to two kinematic phases. Consequently, when interacting with multiple objects, redirecting together in the "reach-to-grasp" and "reach-to-place" phases has the potential to extend the range of multiple virtual objects represented by the same physical object.

Participants' adaptability to redirection. Throughout our study, we noted a progressive improvement in participants' ability to acclimate to the assigned tasks. Additionally, we observed a tendency to adapt to each new redirection setting during repeated iterations gradually. Comparing the two baseline conditions in the second experiment, the scores of the first baseline were slightly worse than the scores of the second baseline in every scale, likely because the participants became more adapted to the experiment environment as the experiment continued. Furthermore, another one-way ANOVA was carried out to investigate whether any differences appeared in participants' motions across multiple trials of the same condition within the specified number of repetitions. Based on the analysis, it was observed that the completion time and movement distance decreased progressively over the five trials. However, neither the completion time nor the total movement distance revealed any statistical difference among the repeated trials, which means that most participants did not fully adapt to the redirection in each of the five replications in our setup. This finding is consistent with the previous argument that participants will probably adapt to redirection within 4-8 times by Feick et al.[9].

Limitations and Future Work. Our work also has some limitations. Firstly, our user study primarily concerns redirection in the horizontal direction. This is because comparing more dimensions will increase the number of trials in the experiment, which can be exhausting for participants. Our evaluation in the horizontal direction shows that the detection thresholds differ for the "reach-to-grasp" and "reach-to-place" phases, and the combination of RR and RP can bring users a better embodiment experience when a larger redirected offset is required. Hand redirection between different kinematics phases should be studied correspondingly in the vertical and depth directions. Additionally, the participants have an uneven gender distribution in experiments 1 and 2. A few works suggest noticeable differences in low-level perceptual tasks between the genders [32]. Although the variation of detection thresholds using RP, RP, and RR & RP methods keeps consistent between genders, we observed distinct detection thresholds for male ($DT_{RP} = 18.97cm > DT_{RR\&RP} = 13.44cm > DT_{RR} = 12.97cm$) and female ($DT_{RP} = 14.70cm > DT_{RR\&RP} = 10.74cm > DT_{RR} = 10.13cm$) subjects. Future work may further explore the gender-adapted RR and RP hand redirection methods. Moreover, when we design the RP approach, we simplify the problem to script the positions of the virtual objects to be placed in advance and with only two virtual objects. VR training applications with a pre-defined interaction sequence and little difference in shape and weight between multiple interactive objects are most likely to benefit from the proposed combined RR&RP method. However, it would be more natural to allow the users to put the objects arbitrarily. Future work on predicting the placement of the objects can make the interaction more flexible, and complex tasks with more objects will considerably increase the versatility of the proposed method in practical applications.

6 CONCLUSION

In conclusion, we proposed *Redirected Placement* (RP) that can be utilized to accurately redirect the placement of physical props to a specific location during the "reach-to-place" kinematic phase. The detection thresholds in the "reach-to-grasp" and the "reach-to-place" phases are estimated in the first experiment, indicating the higher threshold for the latter phase. We further conducted the second experiment for evaluating the embodiment experience when using hand redirection with RR-only, RP-only, and RR&RP with various combination ratios. The result reports the best sense of embodiment in the condition of RR and RP equally combined. Our findings expect to provide a reference for optimizing the user experience when designing hand-redirection-related applications in VR.

ACKNOWLEDGMENTS

The authors thank all participants of the studies for their time and contribution. This research is supported by the National Key Research and Development Program of China (2018YFB1004902).

REFERENCES

[1] Mahdi Azmandian, Mark Hancock, Hrvoje Benko, Eyal Ofek, and Andrew D. Wilson. 2016. Haptic Retargeting: Dynamic Repurposing of Passive Haptics for Enhanced Virtual Reality Experiences. In *Proceedings of the 2016 CHI Conference on Human Factors in Computing Systems*. 1968–1979. https://doi.org/10.1145/2858036.2858226

[2] Brett Benda, Shaghayegh Esmaeili, and Eric D. Ragan. 2020. Determining Detection Thresholds for Fixed Positional Offsets for Virtual Hand Remapping in Virtual Reality. In *2020 IEEE International Symposium on Mixed and Augmented Reality (ISMAR)*. 269–278. https://doi.org/10.1109/ISMAR50242.2020.00050

[3] Joanna Bergström, Aske Mottelson, and Jarrod Knibbe. 2019. Resized Grasping in VR: Estimating Thresholds for Object Discrimination. In *Proceedings of the 32nd Annual ACM Symposium on User Interface Software and Technology*. 1175–1183. https://doi.org/10.1145/3332165.3347939

[4] Eric Burns, Sharif Razzaque, Abigail T. Panter, Mary C. Whitton, Matthew R. McCallus, and Frederick P. Brooks. 2006. The Hand Is More Easily Fooled than the Eye: Users Are More Sensitive to Visual Interpenetration than to Visual-Proprioceptive Discrepancy. *Presence* 15, 1 (2006), 1–15. https://doi.org/10.1162/pres.2006.15.1.1

[5] Lung-Pan Cheng, Eyal Ofek, Christian Holz, Hrvoje Benko, and Andrew D. Wilson. 2017. Sparse Haptic Proxy: Touch Feedback in Virtual Environments Using a General Passive Prop. In *Proceedings of the 2017 CHI Conference on Human Factors in Computing Systems*. 3718–3728. https://doi.org/10.1145/3025453.3025753

[6] Michel Desmurget, Denis Pélisson, Yves Rossetti, and Claude Prablanc. 1998. From eye to hand: planning goal-directed movements. *Neuroscience & Biobehavioral Reviews* 22, 6 (1998), 761–788.

[7] Shaghayegh Esmaeili, Brett Benda, and Eric D. Ragan. 2020. Detection of Scaled Hand Interactions in Virtual Reality: The Effects of Motion Direction and Task Complexity. In *2020 IEEE Conference on Virtual Reality and 3D User Interfaces (VR)*. IEEE, 453–462. https://doi.org/10.1109/VR46266.2020.00066

[8] Cathy Mengying Fang. 2021. Retargeted Self-Haptics for Increased Immersion in VR without Instrumentation. In *Proceedings of the 34th Annual ACM Symposium on User Interface Software and Technology*. 1109–1121. https://doi.org/10.1145/3472749.3474810

[9] Martin Feick, Kora P. Regitz, Anthony Tang, Tobias Jungbluth, Maurice Rekrut, and Antonio Krüger. 2023. Investigating Noticeable Hand Redirection in Virtual Reality using Physiological and Interaction Data. In *2023 IEEE Conference Virtual Reality and 3D User Interfaces (VR)*. 194–204. https://doi.org/10.1109/VR55154.2023.00035

[10] J Randall Flanagan, Miles C Bowman, and Roland S Johansson. 2006. Control strategies in object manipulation tasks. *Current Opinion in Neurobiology* 16, 6 (Dec. 2006), 650–659. https://doi.org/10.1016/j.conb.2006.10.005

[11] Henrique Galvan Debarba, Ronan Boulic, Roy Salomon, Olaf Blanke, and Bruno Herbelin. 2018. Self-attribution of distorted reaching movements in immersive virtual reality. *Computers & Graphics* 76 (Nov. 2018), 142–152. https://doi.org/10.1016/j.cag.2018.09.001

[12] Benoit Geslain, Simon Besga, Flavien Lebrun, and Gilles Bailly. 2022. Generalizing the Hand Redirection Function in Virtual Reality. In *Proceedings of the 2022 International Conference on Advanced Visual Interfaces (AVI 2022)*. 1–9. https://doi.org/10.1145/3531073.3531100

[13] J. J. Gibson. 1933. Adaptation, after-effect and contrast in the perception of curved lines. *Journal of Experimental Psychology* 16, 1 (1933), 1–31. https://doi.org/10.1037/h0074626

[14] Eric J. Gonzalez, Parastoo Abtahi, and Sean Follmer. 2020. REACH+: Extending the Reachability of Encountered-type Haptics Devices through Dynamic Redirection in VR. In *Proceedings of the 33rd Annual ACM Symposium on User Interface Software and Technology*. 236–248. https://doi.org/10.1145/3379337.3415870

[15] Eric J. Gonzalez and Sean Follmer. 2019. Investigating the Detection of Bimanual Haptic Retargeting in Virtual Reality. In *25th ACM Symposium on Virtual Reality Software and Technology*. 1–5. https://doi.org/10.1145/3359996.3364248

[16] Mar Gonzalez-Franco and Jaron Lanier. 2017. Model of Illusions and Virtual Reality. *Frontiers in Psychology* 8 (2017). https://www.frontiersin.org/articles/10.3389/fpsyg.2017.01125

[17] Mar Gonzalez-Franco and Tabitha C. Peck. 2018. Avatar Embodiment. Towards a Standardized Questionnaire. *Frontiers in Robotics and AI* 5 (2018). https://www.frontiersin.org/articles/10.3389/frobt.2018.00074

[18] Dustin T. Han, Mohamed Suhail, and Eric D. Ragan. 2018. Evaluating Remapped Physical Reach for Hand Interactions with Passive Haptics in Virtual Reality. *IEEE Transactions on Visualization and Computer Graphics* 24, 4 (April 2018), 1467–1476. https://doi.org/10.1109/TVCG.2018.2794659

[19] Judith Hartfill, Jenny Gabel, Lucie Kruse, Susanne Schmidt, Kevin Riebandt, Simone Kühn, and Frank Steinicke. 2021. Analysis of Detection Thresholds for Hand Redirection during Mid-Air Interactions in Virtual Reality. In *Proceedings of the 27th ACM Symposium on Virtual Reality Software and Technology*. 1–10. https://doi.org/10.1145/3489849.3489866

[20] Flavien Lebrun, Sinan Haliyo, and Gilles Bailly. 2021. A Trajectory Model for Desktop-Scale Hand Redirection in Virtual Reality. In *Human-Computer-Interaction – INTERACT 2021 (Lecture Notes in Computer Science)*, Carmelo Ardito, Rosa Lanzilotti, Alessio Malizia, Helen Petrie, Antonio Piccinno, Giuseppe Desolda, and Kori Inkpen (Eds.). 105–124. https://doi.org/10.1007/978-3-030-85607-6_8

[21] Lei Liu, Robert van Liere, Catharina Nieuwenhuizen, and Jean-Bernard Martens. 2009. Comparing Aimed Movements in the Real World and in Virtual Reality. In *2009 IEEE Virtual Reality Conference*. 219–222. https://doi.org/10.1109/VR.2009.4811026 ISSN: 2375-5334.

[22] Andreas L. Lohse, Christoffer K. Kjær, Ervin Hamulic, Ingrid G. A. Lima, Tilde H. Jensen, Luis E. Bruni, and Niels C. Nilsson. 2019. Leveraging Change Blindness for Haptic Remapping in Virtual Environments. In *2019 IEEE 5th Workshop on Everyday Virtual Reality (WEVR)*. 1–5. https://doi.org/10.1109/WEVR.2019.8809587

[23] Jaime Maldonado and Christoph Zetzsche. 2021. Object Manipulations in VR Show Task- and Object-Dependent Modulation of Motor Patterns. In *Proceedings of the 27th ACM Symposium on Virtual Reality Software and Technology (VRST '21)*. Association for Computing Machinery, New York, NY, USA, 1–9. https://doi.org/10.1145/3489849.3489858

[24] Brandon J. Matthews, Bruce H. Thomas, G. Stewart Von Itzstein, and Ross T. Smith. 2022. Shape Aware Haptic Retargeting for Accurate Hand Interactions. In *2022 IEEE Conference on Virtual Reality and 3D User Interfaces (VR)*. 625–634. https://doi.org/10.1109/VR51125.2022.00083 ISSN: 2642-5254.

[25] Brandon J. Matthews, Bruce H. Thomas, Stewart Von Itzstein, and Ross T. Smith. 2019. Remapped Physical-Virtual Interfaces with Bimanual Haptic Retargeting. In *2019 IEEE Conference on Virtual Reality and 3D User Interfaces (VR)*. IEEE, 19–27. https://doi.org/10.1109/VR.2019.8797974

[26] Niels Christian Nilsson, André Zenner, and Adalberto L. Simeone. 2021. Propping Up Virtual Reality With Haptic Proxies. *IEEE Computer Graphics and Applications* 41, 5 (Sept. 2021), 104–112. https://doi.org/10.1109/MCG.2021.3097671

[27] Nami Ogawa, Takuji Narumi, and Michitaka Hirose. 2021. Effect of Avatar Appearance on Detection Thresholds for Remapped Hand Movements. *IEEE Transactions on Visualization and Computer Graphics* 27, 7 (July 2021), 3182–3197. https://doi.org/10.1109/TVCG.2020.2964758

[28] Shahina Pardhan, Amy Scarfe, Rupert Bourne, and Matthew Timmis. 2017. A Comparison of Reach-to-Grasp and Transport-to-Place Performance in Participants With Age-Related Macular Degeneration and Glaucoma. *Invest. Ophthalmol. Vis. Sci.* 58, 3 (March 2017), 1560. https://doi.org/10.1167/iovs.16-20273

[29] Cristian Patras, Mantas Cibulskis, and Niels Christian Nilsson. 2022. Body Warping Versus Change Blindness Remapping: A Comparison of Two Approaches to Repurposing Haptic Proxies for Virtual Reality. In *2022 IEEE Conference on Virtual Reality and 3D User Interfaces (VR)*. IEEE, 205–212. https://doi.org/10.1109/VR51125.2022.00039

[30] Mohamed Suhail Mohamed Yousuf Sait, Shyam Prathish Sargunam, Dustin T. Han, and Eric D. Ragan. 2018. Physical hand interaction for controlling multiple virtual objects in virtual reality. In *Proceedings of the 3rd International Workshop on Interactive and Spatial Computing (IWISC '18)*. Association for Computing Machinery, 64–74. https://doi.org/10.1145/3191801.3191814

[31] Majed Samad, Elia Gatti, Anne Hermes, Hrvoje Benko, and Cesare Parise. 2019. Pseudo-Haptic Weight: Changing the Perceived Weight of Virtual Objects By Manipulating Control-Display Ratio. In *Proceedings of the 2019 CHI Conference on Human Factors in Computing Systems (CHI '19)*. Association for Computing Machinery, New York, NY, USA, 1–13. https://doi.org/10.1145/3290605.3300550

[32] Albulena Shaqiri, Maya Roinishvili, Lukasz Grzeczkowski, Eka Chkonia, Karin Pilz, Christine Mohr, Andreas Brand, Marina Kunchulia, and Michael H Herzog. 2018. Sex-related differences in vision are heterogeneous. *Scientific reports* 8, 1 (2018), 7521.

[33] Patrick L. Strandholt, Oana A. Dogaru, Niels C. Nilsson, Rolf Nordahl, and Stefania Serafin. 2020. Knock on Wood: Combining Redirected Touching and Physical Props for Tool-Based Interaction in Virtual Reality. In *Proceedings of the 2020 CHI Conference on Human Factors in Computing Systems*. 1–13. https://doi.org/10.1145/3313831.3376303

[34] Mohamed Suhail, Shyam Prathish Sargunam, Dustin T. Han, and Eric D. Ragan. 2017. Redirected reach in virtual reality: Enabling natural hand interaction at multiple virtual locations with passive haptics. In *2017 IEEE Symposium on 3D User Interfaces (3DUI)*. IEEE, 245–246. https://doi.org/10.1109/3DUI.2017.7893363

[35] Matthew A. Timmis and Shahina Pardhan. 2012. The Effect of Central Visual Impairment on Manual Prehension When Tasked with Transporting-to-Place an Object Accurately to a New Location. *Invest. Ophthalmol. Vis. Sci.* 53, 6 (May 2012), 2812. https://doi.org/10.1167/iovs.11-8860

[36] Xavier de Tinguy, Claudio Pacchierotti, Mathieu Emily, Mathilde Chevalier, Aurélie Guignardat, Morgan Guillaudeux, Chloé Six, Anatole Lécuyer, and Maud Marchal. 2019. How Different Tangible and Virtual Objects Can Be While Still Feeling the Same?. In *2019 IEEE World Haptics Conference (WHC)*. 580–585. https://doi.org/10.1109/WHC.2019.8816164

[37] Jackie (Junrui) Yang, Hiroshi Horii, Alexander Thayer, and Rafael Ballagas. 2018. VR Grabbers: Ungrounded Haptic Retargeting for Precision Grabbing Tools. In *Proceedings of the 31st Annual ACM Symposium on User Interface Software and*

Technology (UIST '18). 889–899. https://doi.org/10.1145/3242587.3242643

[38] Andre Zenner and Antonio Kruger. 2019. Estimating Detection Thresholds for Desktop-Scale Hand Redirection in Virtual Reality. In *2019 IEEE Conference on Virtual Reality and 3D User Interfaces (VR)*. IEEE, 47–55. https://doi.org/10.1109/VR.2019.8798143

[39] André Zenner and Antonio Krüger. 2020. Shifting & Warping: A Case for the Combined Use of Dynamic Passive Haptics and Haptic Retargeting in VR. In *Adjunct Publication of the 33rd Annual ACM Symposium on User Interface Software and Technology (UIST '20 Adjunct)*. 1–3. https://doi.org/10.1145/3379350.3416166

[40] Andre Zenner, Kora Persephone Regitz, and Antonio Kruger. 2021. Blink-Suppressed Hand Redirection. In *2021 IEEE Virtual Reality and 3D User Interfaces (VR)*. IEEE, 75–84. https://doi.org/10.1109/VR50410.2021.00028

[41] Andre Zenner, Kristin Ullmann, and Antonio Kruger. 2021. Combining Dynamic Passive Haptics and Haptic Retargeting for Enhanced Haptic Feedback in Virtual Reality. *IEEE Transactions on Visualization and Computer Graphics* 27, 5 (May 2021), 2627–2637. https://doi.org/10.1109/TVCG.2021.3067777

A LAGRANGIAN FUNCTION FOR PHYSICAL TARGET SELECTION

We use the Lagrange multiplier method to solve the optimization problem in Eq. 1. The Lagrangian function for this problem is:

$$L(\boldsymbol{x}, \lambda) = \frac{1}{2}\omega_1 \|\boldsymbol{x} - \boldsymbol{p}_a\|_2^2 + \frac{1}{2}\omega_2 \|\boldsymbol{x} - \boldsymbol{p}_b\|_2^2 + \lambda(\|\boldsymbol{x} - \boldsymbol{p}_p\|_2^2 - \theta^2). \tag{5}$$

where λ is the Lagrange multiplier. Then, we discuss this problem in two cases: an interior solution with an inactive constraint and a boundary solution with an active constraint.

The solution within the boundary condition is the interpolation point $\boldsymbol{x}_0$ between $\boldsymbol{p}_a$ and $\boldsymbol{p}_b$:

$$\boldsymbol{x} = \boldsymbol{x}_0 = \omega_1 \boldsymbol{p}_a + \omega_2 \boldsymbol{p}_b. \tag{6}$$

Otherwise, if $\|\boldsymbol{x}_0 - \boldsymbol{p}_p\|_2^2 > \theta^2$, the physical prop is relocated from $\boldsymbol{p}_p$ to $\boldsymbol{x}_0$ direction in the magnitude of θ:

$$\boldsymbol{x} = \boldsymbol{p}_p + \theta \frac{(\boldsymbol{x}_0 - \boldsymbol{p}_p)}{\|\boldsymbol{x}_0 - \boldsymbol{p}_p\|_2} \tag{7}$$

B EMBODIMENT QUESTIONNAIRE

The embodiment questionnaire used in our second experiment is a subset of the standardized embodiment questionnaire by Gonzalez-Franco et al. [17].

1. Body (Hand) ownership.

Q1. "I felt as if the virtual hand was my hand."

Q2. "It felt as if the virtual hand I saw was someone else."

Q3. "It seemed as if I might have more than one hand."

2. Agency and motor control.

Q4. "It felt like I could control the virtual hand as if it was my own hand."

Q5. "The movements of the virtual hand were caused by my movements."

Q6. "I felt as if the movements of the virtual hand were influencing my own movements."

Q7. "I felt as if the virtual hand was moving by itself."

3. Location of the body (hand).

Q8. "I felt as if my hand was located where I saw the virtual hand."

Q9. "I felt out of my body."

Q10. "I felt as if my (real) hand were drifting toward the virtual hand or as if the virtual hand were drifting toward my (real) hand."